15 Condenser and Reboiler Dynamics

15.1 LIQUID-COOLED CONDENSERS WITH NO CONDENSATE HOLDUP

As mentioned in Chapter 3, we find in the chemical and petroleum industries two principal types of liquid-cooled condensers: (1) the horizontal type with vapor on the shell side and coolant in the tubes, and (2) the vertical design with vapor in the tubes and coolant on the shell side. We can also think of condensers in terms of whether the coolant goes through just once (no axial mixing) or is recirculated to achieve good axial mixing. A condenser with the latter type of cooling is said to have "tempered" cooling.

Condenser with No Axial Mixing of Coolant

Once-through coolant is by far the most common choice. An approximate analysis for a condenser that has a single pass on the coolant side is presented in Chapter 24 of reference 1, and will not be repeated here. It involves the following simplifying assumptions:

1. Heat storage in the heat exchanger metal is negligible.
2. Subcooling is negligible.
3. Mean temperature difference is arithmetic.

Although these reduce the complexity somewhat, we are still left with the job of solving partial differential equations. Perhaps the easiest to read descriptions of their solutions are those by Hempel\(^2\) and by Gould.\(^3\) A much simpler model, largely empirical, has been proposed by Thal-Larsen.\(^4\) Since most liquid-cooled condensers are fairly fast with time constants in the range of 10–60 seconds, we will not pursue their dynamic equations further. For important applications, where subcooling may be of concern, one should probably resort to simulation.
The literature on subcooled condensers is very sparse; one paper has been published by Luyben, Archambault, and Jauffret, and another by Tyreus.

If the sensible heat load of subcooling is not too large compared with that of the condensing heat load (and this is usually the case), the following static gains may be derived:

\[
\frac{\partial w_c}{\partial w_A} = \frac{(T_c - T_{A1}) \left[ 2 \bar{U}_c A_c^2 c_A + 4 \bar{w}_A^2 c_A A_c \frac{\partial U_c}{\partial w_A} \right]}{\lambda_p (\bar{U}_c A_c + 2 \bar{w}_A c_A)^2}
\] (15.1)

\[
\frac{\partial w_c}{\partial T_{A1}} = \frac{-2 \bar{U}_c A_c \bar{w}_A c_A}{\lambda_p (\bar{U}_c A_c + 2 \bar{w}_A c_A)}
\] (15.2)

\[
\frac{\partial w_c}{\partial T_c} = \frac{2 \bar{U}_c A_c}{\lambda_p (\bar{U}_c A_c + 2 \bar{w}_A c_A)}
\] (15.3)

\[
\frac{\partial T_{AO}}{\partial T_c} = \frac{2 \bar{U}_c A_c}{\bar{U}_c A_c + 2 \bar{w}_A c_A}
\] (15.4)

\[
\frac{\partial T_{AO}}{\partial T_{A1}} = \frac{-2 \bar{U}_c A_c - 2 \bar{w}_A c_A}{\bar{U}_c A_c + 2 \bar{w}_A c_A}
\] (15.5)

\[
\frac{\partial T_{AO}}{\partial w_A} = \frac{4 \bar{w}_A c_A \frac{\partial U_c}{\partial w_A} (T_c - T_{A1}) - 4 \bar{U}_c A_c c_A (T_c - T_{A1})}{(\bar{U}_c A_c + 2 \bar{w}_A c_A)^2}
\] (15.6)

where

- \(T_c\) = process condensing temperature, °K
- \(T_{A1}\) = coolant inlet temperature, °K
- \(T_{AO}\) = coolant exit temperature, °K
- \(c_A\) = coolant specific heat, pcu/lbm °C
- \(U_c\) = condenser heat-transfer coefficient, pcu/sec °C ft
- \(w_c\) = rate of condensation, lbm/sec
- \(w_A\) = coolant flow rate, lbm/sec
- \(\lambda_p\) = latent heat, pcu/lbm, of process vapor
- \(A_c\) = condenser heat-transfer area, ft

Condenser with Well-Mixed Coolant

Qualitatively the condenser with well-mixed (tempered) coolant is discussed in Chapter 3, Section 9 (see Figure 3.19). A mathematical analysis of its dynamics is given in Chapter 24 of reference 1. We will not repeat it here, but it leads to the following results (simplifying assumptions are the same as in the previous section):
15.2 Flooding Condensers—Open-Loop Dynamics

The first-order dynamics of this type of condenser make it much easier to control than the condenser with once-through coolant.
1. The sensible heat load is small enough in comparison with the latent heat load that it may be neglected.

2. Submerged heat-transfer area, $A_s$, is proportional to liquid level above the bottom of the lowest tube. The change in condensing area, $A_c$, is then the negative of the change in $A_s$. This assumption is not bad if there are many tubes and if they are not "layered." To make this assumption valid may require that in some cases the tube bundle be slightly rotated about its axis.

3. For the time being, we will assume that the vent valve position is fixed.

4. Heat storage in the heat exchanger metal may be neglected.

FIGURE 15.1
Horizontal condenser with coolant in tubes and partially flooded on shell side
We may now write the following equations, some in the time domain and some in the $s$ domain:

\begin{align}
q_c(t) &= U_c A_c(t) \Delta T(t) \\
\dot{w}_c(t) &= \frac{q_c(t)}{\lambda_p} \\
\frac{w_c(s) - w_o(s)}{s} &= W_{SH}(s) \\
\dot{w}_w(t) - \dot{w}_v(t) &= \dot{w}_w(t) \\
A_c(s) &= \frac{\partial A_c}{\partial W_{SH}} W_{SH}(s) \\
A_c(s) &= -A_c(s) \\
\Delta T(s) &= T_c(s) - T_A(s) \\
T_c(s) &= \frac{\partial T_c}{\partial \rho_c} P_c(s) \\
\frac{P_c(s) - P_R(s)}{R_v} &= \frac{\dot{w}_w(s)}{\rho_v} \\
&= \frac{\sum Q(s)/(V/P_c)s}{V}
\end{align}

where

- $V =$ system vapor volume, ft\(^3\)
- $q_c =$ rate of heat transfer, pcu/sec
- $U_c =$ condenser heat-transfer coefficient, \(\text{pcu/sec ft}^2^\circ\text{C}\)
- $A_c =$ condensing heat-transfer area, ft\(^2\)
- $T =$ \(^\circ\text{C}\) or \(^\circ\text{K}\)
- $w_c =$ lbm/sec process vapor condensed
- $\lambda_p =$ latent heat of vaporization, pcu/lbm, of condensing process vapor
- $w_o =$ liquid outflow, lbm/sec
- $w_v =$ vapor inflow, lbm/sec
- $w_w =$ vapor outflow, lbm/sec
- $A_s =$ submerged heat-transfer area, ft\(^2\)
- $T_c =$ condensing temperature, \(^\circ\text{C}\) or \(^\circ\text{K}\)
- $T_A =$ average coolant temperature, \(^\circ\text{C}\) or \(^\circ\text{K}\)
- $P_c =$ vapor space pressure, lbf/ft\(^2\)
- $P_R =$ pressure, lbf/ft\(^2\), downstream of vent valve
- $R_v =$ vent valve resistance, lbf sec/ft\(^5\)
- $W_{SH} =$ pounds of liquid in shell between bottom of lowest tube and top of highest tube
- $A_{TC} =$ total heat-transfer area, ft\(^2\), of condenser
- $Q =$ flow, ft\(^3\)/sec
Upon Laplace transforming equations (15.13), (15.14), and (15.16), we obtain:

\[ q_c(s) = U_c \Delta T(s) + U_c \Delta T A_c(s) \]  

(15.22)

\[ w_c(s) = \frac{q_c(s)}{\lambda_p} \]  

(15.23)

\[ w_w(s) - w_c(s) = w_w(s) \]  

(15.24)

**Response, \( P_c(s) \), to Various Inputs**

To find the response, \( P_c(s) \), to various inputs, we combine equations (15.13) through (15.24) into the signal flow diagram of Figure 15.2. The first reduction of this is given in Figure 15.3 and the final reduction in Figure 15.4. From this we may write by inspection:

\[
P_c(s) = \left[ \frac{R_s/\rho_r}{s + \alpha} \right] \frac{\frac{V}{P_e} R_s}{s^2 + \frac{1 + \alpha R_s \frac{V}{P_e} + K_{CO}}{\alpha} + \frac{1}{s + 1}} \times \left[ \frac{-\alpha}{s + \alpha} w_a(s) + \frac{U_c A_c s/\lambda_p}{s + \alpha} T_A(s) + \frac{\rho_r}{\lambda_p} P_R(s) + w_w(s) \right] \]  

(15.25)

where

\[ \alpha = \frac{\partial A_s}{\partial W_{SH}} U_c \Delta T / \lambda_p \]

\[ K_{CO} = \frac{R_s/\rho_r}{\lambda_p} \frac{\partial T_C}{\partial P_e} U_c A_s \]

The first term on the right-hand side of equation (15.25) may be written:

\[
\frac{K_{FC} (s + \alpha)}{\tau_{Q}^2 s^2 + 2 \zeta \tau_{Q} s + 1}
\]

For all such condensers we have studied, the denominator has a large damping ratio so that the quadratic may be factored into two terms, one of which has a typical time constant of 1.5-3 minutes, while the other is only a few seconds. The above then reduces to:

\[
\frac{K_{FC} (s + \alpha)}{\tau_{FC} s + 1}
\]

where

\[ \tau_{FC} = \frac{1 + \alpha R_s \frac{V}{P_e} + K_{CO}}{\alpha} \]
FIGURE 15.2
First signal flow diagram for $P_c$ of flooded condenser
FIGURE 15.3
First reduction of signal flow diagram of figure 15.2
Final signal flow diagram for $P_c$ of flooded condenser

\[
\frac{R_u/\rho_v}{\left(\frac{V}{P_c}\right) R_v s + 1} \\
1 + \left(\frac{V}{P_c}\right) R_v s + 1 \\
\frac{\frac{\partial T_c}{\partial P_c} U_c \bar{A}_c \frac{s}{\lambda_p}}{s + \alpha}
\]
Case Where $R_v$ Approaches Infinity

For the case where $R_v$ becomes very large (very little vent gas flow), the last equation reduces to:

$$P_v(s) = \frac{(1/\rho_v) (s + \alpha)}{\left(\alpha \frac{V}{P_c} + \frac{U_c A_c \partial T_{ce}}{\lambda_p \rho_v \partial P_c}\right) s \left(\frac{V}{P_c} + \frac{U_c A_c \partial T_{ce}}{\lambda_p \rho_v \partial P_c} s + 1\right)}$$

$$\times \left[ -\frac{\alpha}{s + \alpha} w_v(s) + \frac{U_A e/\lambda_p}{s + \alpha} T_A(s) + w_n(s) \right] \quad (15.26)$$

The first term on the right may be written:

$$\frac{K'_f_c (s + \alpha)}{s(\tau'_{f_c} s + 1)}$$

where

$$K'_{f_c} = \frac{1/\rho_v}{\alpha \frac{V}{P_c} + \frac{U_c A_c \partial T_{ce}}{\lambda_p \rho_v \partial P_c}}$$

and

$$\tau'_{f_c} = \frac{\frac{V}{P_c}}{\alpha \frac{V}{P_c} + \frac{U_c A_c \partial T_{ce}}{\lambda_p \rho_v \partial P_c}}$$

Response, $w_e(s)$, to Various Inputs

The signal flow diagram of Figure 15.2 can be redrawn to show $w_e(s)$ as an output as shown in Figure 15.5. This can then be reduced to the form of Figure 15.6, from which we can write by inspection:

$$w_e(s) = \Omega(s) \left(\frac{\alpha \lambda^p_e}{s} w_n(s) - U_c A_c T_A(s) + \left[\frac{(R_v/\rho_v) U_c A_c \partial T_{ce}}{\partial P_c} \frac{V}{P_c R_v s + 1}\right] \right.$$  

$$\times \left[ w_v(s) + \frac{\rho_v}{R_v} P_R(s) \right] \quad (15.27)$$
where
\[
\Omega(s) = \frac{s}{\alpha \lambda_p \left( \frac{V}{P_c} R_p s + 1 \right)} \left[ \frac{V}{P_c} R_p s^2 + \frac{1 + \alpha \frac{V}{P_c} R_p + K_{CO}}{\alpha} s + 1 \right] \nonumber
\]

\[
= \frac{K_{FC}^2 \left( \frac{V}{P_c} R_p s + 1 \right)}{\tau_Q^2 s^2 + 2 \xi \tau_Q s + 1}
\] (15.27a)

**Case Where \( R_p \) Approaches Infinity**

Again, as \( R_p \) becomes very large, the preceding equation becomes simpler:

\[
w_c(s) = \left[ \frac{(V/P_c \lambda_p)}{\alpha V/P_c + \frac{U_c \Delta \theta_c}{\rho_p \lambda_p} \frac{\delta_t}{\delta P_c}} \times \frac{s}{\alpha V/P_c + \frac{U_c \Delta \theta_c}{\rho_p \lambda_p} \frac{\delta_t}{\delta P_c}} + \frac{1}{\frac{V}{P_c} s} \right] \nonumber
\]

\[
\times \left[ \frac{\alpha \lambda_p}{s} w_o(s) - \frac{U_c \Delta \theta_c}{\rho_p \frac{V}{P_c}} T_A(s) + \frac{1}{\rho_p \frac{V}{P_c}} w_{\text{in}}(s) \right]
\] (15.28)

### 15.3 REBOILERS—OPEN-LOOP DYNAMICS

We wish to make an analysis of column bases with associated reboilers where there is significant liquid holdup. The analysis should take into account the temperature of the entering liquid and the sensible heat effect of the liquid mass. Such an analysis also applies to vaporizers with associated separators or knockout drums.

**Heat-Transfer Dynamics**

The combination of thermosyphon reboiler (or any high circulation rate reboiler) and column base or separator may be represented as shown schematically in Figure 15.7. The various equations may then be written as follows.

**Heat Balance**

\[
w_i(t) c_p T_i(t) + q_T(t) - (w_{BU}(t) \left[ \lambda_p + c_p T_{BU}(t) \right]) + w_B(t) c_p T_{BU}(t)) = \frac{dW_B(t) c_p T_{BU}(t)}{dt}
\] (15.29)
FIGURE 15.5
First signal flow diagram for $w_c$ of flooded condenser
FIGURE 15.6
Reduced signal flow diagram for $w_c$ of flooded condenser
By using perturbation techniques and Laplace transforming this equation we get:

\[
c_p \bar{T}_i \bar{w}_i(s) + c_p \bar{w}_i \bar{T}_i(s) + q_T(s) - \lambda_p \bar{w}_{BU}(s) - c_p \bar{w}_{BU} \bar{T}_{BU}(s) - c_p \bar{T}_{BU} \bar{w}_{BU}(s)
- c_p \bar{w}_B \bar{T}_{BU}(s) - c_p \bar{T}_{BU} \bar{w}_B(s) = c_p s \left[ \bar{T}_{BU} \bar{W}_B(s) + \bar{W}_B \bar{T}_{BU}(s) \right]
\]

(15.30)

\[
\int \left[ w_i(t) - w_{BU}(t) - w_B(t) \right] dt = W_B(t)
\]

or

\[
\left[ w_i(s) - w_{BU}(s) - w_B(s) \right] \frac{1}{s} = W_B(s)
\]

(15.31)  (15.32)

**Material Balance**

**Steam-Side Dynamics**

The following are taken from Chapter 25 of reference 1:

\[
T_{cs}(s) = \frac{\partial T_{cs}}{\partial P_{cs}} P_{cs}(s)
\]

(15.33)

\[
P_{cs}(s) = \frac{Q_i(s) - Q_o(s)}{C_R s}
\]

(15.34)

\[
Q_i(s) = \frac{\partial Q_v}{\partial P_v} P_i(s) + \frac{\partial Q_v}{\partial P_{cs}} P_{cs}(s) + \frac{\partial Q_v}{\partial X_v} X_v(s)
\]

(15.35)

\[
Q_o(s) = \frac{q_T(s)}{\lambda_x \rho_x}
\]

(15.36)

\[
q_T(s) = U_R A_R [T_{cs}(s) - T_{BU}(s)]
\]

(15.37)

(Note that heat storage of the reboiler metal is neglected.)

![FIGURE 15.7](image)

Schematic representation of column base and reboiler holdup
In these equations:

\[ T_a(s) = \text{steam condensing temperature, } ^\circ\text{C} \]
\[ P_a(s) = \text{reboiler shell pressure, lbf/ft}^2 \]
\[ Q_s = \text{steam flow rate, ft}^3/\text{sec} \]
\[ q_T = \text{heat transfer, pcu/sec} \]
\[ U_R = \text{reboiler heat-transfer coefficient, } \frac{\text{pcu/}^\circ\text{C}}{\text{ft}^2 \text{ sec}} \]
\[ A_R = \text{heat-transfer area, ft}^2 \]
\[ T_{BU} = \text{boiling temperature of process fluid, } ^\circ\text{C} \]
\[ Q_s = \text{rate of steam condensation, ft}^3/\text{sec} \]
\[ \lambda_\pi = \text{steam latent heat of condensation, pcu/lbm} \]
\[ \rho_\pi = \text{steam density lbm/ft}^3 \text{ at } P_{CS} \text{ and } T_{CS} \]
\[ C_R = \text{acoustic capacitance of reboiler shell, ft}^5/\text{lbf} \]
\[ = V_s/\bar{P}_a \text{ where } V_s = \text{reboiler shell volume, ft}^3 \]
\[ P_s = \text{steam supply pressure, lbf/ft}^2, \text{ upstream of control valve} \]
\[ X_v = \text{valve stem position} \]

The terms \( \partial Q_s/\partial P_a, \partial Q_s/\partial P_{cs}, \) and \( \partial Q_s/\partial X_v \) may be evaluated by the methods of Chapter 15 of reference 1.

The column-base pressure dynamics may be represented by:

\[ P_B(s) = \frac{1}{\rho_{BU}} Z_{\text{col}}(s) w_{BU}(s) \quad (15.38) \]

where

\[ Z_{\text{col}} = \text{column acoustic impedance, looking up from the base, lbf sec/ft}^5 \]
\[ \rho_{BU} = \text{density of vapor boilup, lbm/ft}^3 \]
\[ w_{BU} = \text{rate of boilup, lbm/min} \]
\[ P_B = \text{column-base pressure, lbf/ft}^2 \]

From the preceding equations we can prepare the signal flow diagram of Figure 15.8. This can be partially reduced as shown on Figure 15.9 where three new functions are defined:

\[ \gamma(s) = \frac{1}{\lambda_p + c_p \bar{T}_{BU}} \left[ \lambda_p + c_p \bar{T}_{BU} \right]^{-1} \left[ \frac{\partial T_a Z_{\text{col}}}{\partial P_a \rho_{BU}} \right] \quad (15.39) \]

\[ \phi(s) = \frac{U_R A_R}{1 + U_R A_R \gamma(s) \frac{\partial T_a Z_{\text{col}}(s)}{\partial P_a \rho_{BU}}} \quad (15.40) \]

\[ \Delta(s) = \frac{U_R A_R \gamma(s) \frac{\partial T_a Z_{\text{col}}(s)}{\partial P_a \rho_{BU}}}{1 + U_R A_R \gamma(s) \frac{\partial T_a Z_{\text{col}}(s)}{\partial P_a \rho_{BU}}} \times \frac{1}{\rho_{\pi} \lambda_\pi} \quad (15.41) \]
For all reboilers examined to date, $\Delta(s)$ has been so small, both statically and dynamically, as to be negligible. It therefore will be omitted in the remainder of this book. It is advisable, however, to calculate $\Delta(s)$ for any new system as a check.

We have also found that the sensible heat effect of the liquid mass in the column base or separator is small. Heat-transfer lags are typically only several seconds; vapor flow from the separator follows steam flow almost instantaneously.

---

**FIGURE 15.8**

Preliminary signal flow diagram for heat transfer dynamics
15.3 Reboilers—Open-Loop Dynamics

Base Level Control Cascaded to Steam Flow Control

We assume here that averaging level control is desired. We may then prepare the signal flow diagram of Figure 15.10. Note that:

\[ K_{cf} G_{cf}(s) = \text{flow controller transfer function} \]

\[ K_{mb} = \text{liquid-level transmitter gain} \]

\[ K_{cb} G_{cb}(s) = \text{level controller transfer function} \]

\[ A_B = \text{cross-sectional area of column base, ft}^2 \]

\[ \rho_L = \text{liquid density, lbm/ft}^3 \]

Since \( T_i \) varies slowly or not at all, \( T_i(s) = 0 \).

Since \( T_i \) varies slowly or not at all, \( T_i(s) = 0 \).

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**FIGURE 15.9**
Partial reduction of figure 15.8
Noncritical Versus Critical Steam Flow

If steam flow is critical, then:

$$\frac{1}{1 - \psi(s) \frac{\partial Q_v}{\partial P_{cs}}} = 1$$  \hspace{1cm} (15.42)

since

$$\frac{\partial Q_v}{\partial P_{cs}} = 0$$

Flow control loop gain and dynamics are determined entirely by the instrument characteristics. Note that:

$$\psi(s) = \frac{1}{1 + \frac{1}{C_R s \lambda_\pi \rho_\pi} \phi(s) \frac{\partial T_{cs}}{\partial P_{cs}}}$$  \hspace{1cm} (15.43)

**FIGURE 15.10**

Signal flow diagram for base level control cascaded to steam flow control.
If steam flow is noncritical, 

\[ \frac{1}{1 - \psi(s) \frac{\partial Q_v}{\partial P_a}} \]

has lead characteristics and a static gain of less than unity, permitting higher loop gain, faster flow-control response, and much higher flow controller gain. To avoid problems with flow controller tuning, the system should always be operated in one flow regime or the other. In most cases steam flow is noncritical.

**Signal Flow Diagram Simplification**

Since we have called for averaging level control, the natural frequency of the level control loop will be much lower than that of the flow control loop. Note that:

\[
K_v G_v(s) K_{ef} G_{ef}(s) \frac{\partial Q_v}{\partial X_v} B(s)
\]

\[
= \frac{K_v G_v(s) K_{ef} G_{ef}(s) \frac{\partial Q_v}{\partial X_v} \left( \frac{1}{1 - \psi(s) \frac{\partial Q_v}{\partial P_a}} \right) K_{mf} G_{mf}(s)}{1 + K_v G_v(s) K_{ef} G_{ef}(s) \frac{\partial Q_v}{\partial X_v} \left( \frac{1}{1 - \psi(s) \frac{\partial Q_v}{\partial P_a}} \right) K_{mf} G_{mf}(s)}
\]

\[
\times \frac{1}{K_{mf} G_{mf}(s)} \approx \frac{1}{K_{mf}}
\]

(15.44)

This leads us to the final signal flow diagram of Figure 15.11.

**Base Level Control by Direct Manipulation of Steam Valve**

From Figure 15.9 and by assuming \( \Delta(s) = 0 \), we can prepare the signal flow diagram of Figure 15.12. Note that if steam flow is critical, \( \frac{\partial Q_v}{\partial P_a} = 0 \).

**Further Mathematical Simplification**

It has already been indicated that for averaging level control cascaded to steam flow control, one may substitute \( 1/K_{mf} \) for the flow control loop. Use of the mathematical models discussed here on commercial reboilers indicates that typical time constants range from a fraction of a second to 5–10 seconds. Practically speaking \( \gamma(s) \), \( \phi(s) \), and \( \Delta(s) \) reduce to constants. As we will see in Chapters 16 and 17, it usually will be possible to use much simpler reboiler models than that discussed in this section.
15.4 PARTIALLY FLOODED REBOILERS

The partially flooded reboiler is similar in many ways to the partially flooded condenser. Usually, although not always, it is a vertical thermosyphon reboiler. As discussed in Chapter 4, Section 2, it is controlled by throttling the steam condensate, which in turn varies the condensate level in the shell and thereby the heat-transfer area for condensation. That area covered by liquid permits only sensible heat transfer from the condensate; this is a small heat load compared with that of the condensing steam and is treated as negligible.

The signal flow diagram of Figure 15.5 for a flooded condenser may be used as a starting point. In this case pressure dynamics are essentially negligible. If the steam supply pressure is constant, the steam condensing temperature is also constant.

\[
\frac{\partial Q_v}{\partial P_s} B(s) \psi(s) \frac{\partial T_{ca}}{\partial P_{ca}} \phi(s)
\]

\[
\frac{K_{mh}K_{ch} G_{ca}(s)}{\rho_l A_{bs}} \times \frac{1}{K_{mf}} \times \psi(s) \frac{\partial T_{ca}}{\partial P_{ca}} \phi(s)
\]

\[
c_p(T_l - T_{BU}) + \frac{K_{mh}K_{ch} G_{ca}(s)}{\rho_l A_{bs}} \times \frac{1}{K_{mf}} \psi(s) \frac{\partial T_{ca}}{\partial P_{ca}} \phi(s)
\]

\[
\beta(s)
\]

\[
\frac{\gamma(s)}{1 + c_p T_{BU} \gamma(s)}
\]

\[
\frac{1 + \frac{K_{mh}K_{ch} G_{ca}(s)}{\rho_l A_{bs}} \times \frac{1}{K_{mf}} \times \psi(s) \frac{\partial T_{ca}}{\partial P_{ca}} \phi(s)}{1 + c_p T_{BU} \gamma(s)}
\]

\[
\gamma(s)
\]

\[
\frac{1 + c_p T_{BU} \gamma(s)}{1 + c_p T_{BU} \gamma(s)}
\]

\[
w_{BU}(s)
\]

FIGURE 15.11
Final signal flow diagram for base level control cascaded to steam flow control.
The signal flow diagram of Figure 15.13 may be prepared next. As shown by the reduced form of Figure 15.14:

$$
\frac{w_c(s)}{w_o(s)} = \frac{1}{\frac{\lambda_H}{U_R \Delta T} \frac{\partial A_R}{\partial W_{SH}} + 1}
$$

(15.45)

where

- $w_c$ = rate of steam condensation, lbm/min
- $w_o$ = rate of steam condensate withdrawal, lbm/min
- $\lambda_H$ = steam latent heat, pcu/lbm
- $U_R$ = heat-transfer coefficient, \( \text{pcu/K} \cdot \text{C}/(\text{ft}^2 \cdot \text{min}) \)
- $A_R$ = average exposed heat-transfer area, \( \text{ft}^2 \), for steam condensate
- $\frac{\partial A_R}{\partial W_{SH}} = \frac{A_{RT}}{[W_{SH}]_{\max}}$
- $A_{RT}$ = total heat-transfer area of reboiler, \( \text{ft}^2 \)
- $W_{SH}$ = condensate on shell side, lbm

**FIGURE 15.12**
Signal flow diagram for base level control by direct manipulation of steam valve
FIGURE 15.13
Preliminary signal flow diagram for flooded reboller
FIGURE 15.14
Reduced signal flow diagram for flooded reboiler
Design experience with flooded reboilers is limited but indicates that typical time constants are of the order of 2–5 minutes. Simulation studies show that substantial improvement in response speed may be achieved by lead–lag compensations with transfer functions such as:

$$\frac{\tau_D s + 1}{\alpha s + 1}$$

where we let:

$$\tau_D = \frac{\lambda_R}{U_R \Delta T \frac{dA_R}{dW_{SH}}} = \tau_{RB} \quad (15.46)$$

Commercial lead–lag compensators commonly have values of $\alpha$ between 6 and 30. Some provide a fixed $\alpha$ and some have adjustable $\alpha$. The former are usually much less expensive.

It is interesting to look at the response of $w_c$ to a change in $\Delta T$. From Figure 15.14:

$$\frac{w_c(s)}{\Delta T(s)} = U_R \bar{A}_R \left[ \frac{\tau_{RB} s}{\lambda_R} \times \frac{1}{\tau_{RB} s + 1} \right] \quad (15.47)$$

If $w_c$ were held constant, we would intuitively expect a step increase in $\Delta T$ to cause an initial increase in heat transfer, and therefore in $w_c$. Condensate level, however, would eventually increase (thereby decreasing $A_R$) and eventually $w_c$ would have to equal $w_e$. Solving equation (15.47) for a step increase in $\Delta T$ shows this to be true:

$$\Delta w_c(t) = \mathcal{L}^{-1} \left[ \frac{\Delta (\Delta T)}{s} U_R \bar{A}_R \left[ \frac{s}{\lambda_R} \times \frac{1}{s + \frac{1}{\tau_{RB}}} \right] \right] \quad (15.48)$$

so that:

$$\Delta w_c(t) = \Delta (\Delta T) \left( \frac{U_R \bar{A}_R}{\lambda_R} \right) e^{-t/\tau_{RB}} \quad (15.49)$$

In equation (15.47) $w_c(t)$ is a perturbation variable (deviation from steady state). As shown by equation (15.49), it decays to zero as time goes to infinity.
15.5 PARTIALLY FLOODED REBOILERS FOR LOW-BOILING MATERIALS

In Chapter 4, Section 2, we discussed a variation in flooded reboiler design for low-boiling materials. As shown by Figure 4.4, we throttle the steam instead of the condensate. The static force–balance relationship is given by equation (4.1):

\[ H_L \rho_L \frac{\rho_L}{g_c} = H_S \rho_L \frac{\rho_L}{g_c} + P_c + \Delta P_{\text{line}} \]

where

- \( H_L \) = loop seal or standpipe height, feet
- \( H_S \) = condensate level in the shell, feet
- \( P_c \) = steam pressure in shell, lbf/ft\(^2\) abs
- \( \rho_L \) = condensate density, lbm/ft\(^3\)

The vented loop seal maintains \( P_c \) just a little above atmospheric pressure.

We can now write the following equations:

\[ P_{\alpha}(s) = \frac{w_c(s) - w_c(s)}{s \rho_c (V/P_c)} \]  
\[ T_\alpha(s) = \frac{\partial T_\alpha}{\partial P_c} P_\alpha(s) \]

From equation (4.1), if we assume that the line loss \( \Delta P_{\text{line}} \) is negligible, and that \( H_L \rho_L \frac{\rho_L}{g_c} \) is constant, then:

\[ H_\alpha(s) = \frac{-P_\alpha(s)}{\rho_c (\rho_L/g_c)} \]

\[ \frac{\partial A_R}{\partial H_\alpha} = \frac{A_{RT}}{[H_\alpha]_{\text{max}}} \]

\[ A_R(s) = \frac{\partial A_R}{\partial H_\alpha} H_\alpha(s) \]

\[ q_T(t) = U_R A_R(t) \Delta T(t) \]

\[ \Delta T = T_\alpha - T_{BU} \]

\[ w_c = \frac{q_T}{\lambda_n} \]

\[ w_{BU} = \frac{q_T}{\lambda_{BU}} \]
FIGURE 15.15
Signal flow diagram for flooded reboiler for low boiling point materials

FIGURE 15.16
Reduced signal flow diagram for flooded reboiler for low boiling point materials
where

\[ w_i = \text{steam flow, lbm/min} \]
\[ w_c = \text{rate of steam condensation, lbm/min} \]
\[ \rho_s = \text{steam density in shell, lbm/ft}^3 \]
\[ V = \text{average free volume in shell above liquid level, ft}^3 \]
\[ T_o = \text{condensing temperature, } ^\circ\text{C} \]
\[ \rho_L = \text{condensate density, lbm/ft}^3 \]

Other terms are as defined earlier in this chapter.

To get equation (15.55) into the \( s \) domain, write:

\[
q_T(s) = \frac{\partial q_T}{\partial A_R} A_R(s) + \frac{\partial q_T}{\partial \Delta T} \Delta T(s) \tag{15.59}
\]

where

\[
\frac{\partial q_T}{\partial A_R} = U_R \Delta T \tag{15.60}
\]
\[
\frac{\partial q_T}{\partial \Delta T} = U_R \Delta \bar{A}_R \tag{15.61}
\]

We may now prepare the signal flow diagram of Figure 15.15, which may then be reduced to the form of Figure 15.16. From the latter we see that:

\[
\frac{w_c(s)}{w_i(s)} = \frac{1}{\lambda \rho_s (\bar{V}/\bar{P}_o) s + 1} \tag{15.62}
\]

where

\[
K_R = \frac{\partial T_o}{\partial P_o} U_R \Delta \bar{A}_R - \frac{\partial A_R/\partial H}{\rho_L} U_R \Delta T \tag{15.63}
\]

Also:

\[
\frac{w_c(s)}{T_{BU}(s)} = \frac{-U_R \Delta \bar{A}_R}{K_R \rho_s (\bar{V}/\bar{P}_o) \times \frac{s}{\tau_{RB}} s + 1} \tag{15.64}
\]

As in the previous section, if there were a step change in \( T_{BU} \), there would be an immediate spike in \( w_c \), which would slowly decay to zero.

REFERENCES


