WEEK 9  ROOT LOCUS PLOTTING

ROOT LOCUS PLOTTING PREPARATION FOR MATLAB

For a FOPDT system, the system transfer function (TF) is

\[ G(s) = \frac{K e^{-t_0 s}}{\tau s + 1} \]

Using Padé's approximation for the exponential term, the TF becomes

\[ G(s) = \frac{K \left( 1 - \frac{t_0}{2} s \right)}{\left( 1 + \frac{t_0}{2} s \right)(\tau s + 1)} = \frac{-K \frac{t_0}{2} s + K}{\frac{t_0 \tau}{2} s^2 + \left( \frac{t_0}{2} + \tau \right)s + 1} \]

For a proportional (only) controller, the controller transfer function is

\[ G_C(s) = K_C \]

So, the OLTF for a FOPDT with proportional control becomes

\[ OLTF = K_C \cdot \frac{-K \frac{t_0}{2} s + K}{\frac{t_0 \tau}{2} s^2 + \left( \frac{t_0}{2} + \tau \right)s + 1} \]

What you put into MATLAB is

\[ num = \begin{bmatrix} -K \frac{t_0}{2} & K \end{bmatrix} \]

and

\[ den = \begin{bmatrix} \frac{t_0 \tau}{2} & \left( \frac{t_0}{2} + \tau \right) & 1 \end{bmatrix} \]

For a proportional-integral controller, the controller transfer function is

\[ G_C(s) = K_C \left( 1 + \frac{1}{\tau I s} \right) = K_C \left( \frac{\tau I s + 1}{\tau I s} \right) \]
So, the OLTF for a FOPDT with proportional integral control becomes

\[
OLTF = K_C \cdot \frac{\left(-K \frac{t_0}{2} \tau_I s^2 + K \left(\tau_I - t_0 \frac{1}{2}\right)s + K\right)}{\left[\tau_I \frac{t_0 \tau}{2} s^3 + \tau_I \left(\frac{t_0}{2} + \tau\right)s^2 + \tau_I s\right]}
\]

What you put into MATLAB is

\[
num = \left[\begin{array}{ccc}
-K \frac{t_0}{2} \tau_I & K \left(\tau_I - t_0 \frac{1}{2}\right) & K
\end{array}\right]
\]

and

\[
den = \left[\begin{array}{ccc}
\tau_I \frac{t_0 \tau}{2} & \tau_I \left(\frac{t_0}{2} + \tau\right) & \tau_I & 0
\end{array}\right]
\]
MATLAB for Windows use

Turn on Computer, right side, back
If necessary, turn on monitor, front, bottom
Wait
Type `win` (this is the software code for "Windows")
Find the MATLAB folder and double click on it, then double click on MATLAB
Type `demo` (this gives you choices of MATLAB demonstrations)
Type `1` (this chooses the "Introduction to basic MATLAB commands" part of the demonstration)
Go through the various information screens, reading and digesting as much as you can. Touch any key to go from one information screen to the next one.
Type `2` (this chooses the "Graphics" part of the demonstration)
Type `9` (this chooses the "Interesting Plots!" part of the demonstration)
Go through the various plot screens. Touch any key to go from one plot screen to the next one.
Type `0` (this chooses to "Quit" the demonstration)
Type `help rlocus` (this lets MATLAB tell you how it does root locus calculations)

Notice it needs `num`, the numerator of the OLT$F$ `without the $K_C$ in it`, and `den`, the denominator of the OLT$F$. So let's enter those.

Let's do an example that is

$$\text{OLTF} = \frac{0.5 \, K_C}{0.5 \, s^2 + 1.5 \, s + 1}$$

Type `num = 0.5` (Leave out the $K_C$)
Type `den = [0.5 1.5 1]` (We enter the coefficients of the polynomial, with spaces between them, in order of the descending power of $s$. You have to put the square brackets around it.)
Type `k = 0:0.5:10` (We tell MATLAB we want to calculate the root locus for values of $K_C$ from 0 (zero) to 10 in steps of 0.5)
Type `r = rlocus (num, den, k)` (This tells MATLAB to do it. The results are in a table named "r")
Type `plot (r,'o')` (This tells MATLAB to make a plot of the results table.)
Type `title('your name ')` (This puts a title on the graph)

Connect the printer to your computer with the printer selector switch.
Bring the plot window to the front; choose "Print" from the "File" menu. (This puts the plot on the paper in the printer.)
You can also "Copy" the graphs or data output and "Paste" into Microsoft Word.

Include root locus plots for your system in the reports for weeks 10, 12 and 14
A draft of Week 10 Report is due the second school day before the next scheduled lab meeting.

**Introduction**

**Theory & Background**
Description & explanation of system components & connections
Schematic diagram
Input function(s) and output functions
Theory & governing equations for components, system and proportional-only feedback controller
Time domain and Laplace domain descriptions, OLTf, CLTF, characteristic equations, $K_{cu}$, $\omega_u$

Quarter decay tuning parameters
Block diagram
Previous system results (gain, time constant, etc.)
Root locus

**Modelling**
Equations & methods used in modelling

**Results**
Modelling results for proportional-only control ($K_{cu}$, quarter decay and critical damped tuning parameters, s.s. error and its dependence on $K_c$)
Design results for system performance with control

**Discussion**
Comparison of theory and modelling of system response with feedback control

**Conclusions**
Values of $K_c$ for specified system response

**Recommendation**

**Appendices**
Physical properties
Modelling diagram, equations

**Attachments**
Include a sheet for each team member that describes the contribution to the work in the laboratory since last reported.

Disk File Suggestion: Use file names beginning with "WR10"